FORMATION OF SIGNALS HARMONIZED WITH THE CHANNEL OF CONNECTION WITH DIFFERENT QUALITY CRITERIA

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Summary The paper presents the optimization of the transmitted signal form according to a number of quality energy criteria. It is the transmission efficiency that has been assessed for signals harmonized with the channel of connection in comparison to the common-used signals.

1. INTRODUCTION

The noise-resistance is one of the most important features determining the efficiency of a radio communication system. It is known that the system noise-resistance is defined both by the transmitted signals form and the kind of their modulation as they can be optimized, i.e. harmonized with a certain channel of connection. The optimization of the transmitted signals form has been studied by many scientists [4] but the problem has been examined mainly from the view point of signal optimal receiving. The issue of their optimal formation in the transmitter has been less investigated. The results obtained have been limited to an evidence of optimal condition of the opposing and simplex signals with transmitting discrete information at a background of Gauss noise. While optimal receiving of signals for such channels is well examined, the problems of harmonizing the transmitted signals form with the channels with different criteria of quality are less developed

The problem of synthesis of signals harmonized with the channel of connection is a variation problem connected with the search of an extreme of the functional used to specify the quality synthesis criterion. The signals searched for are determined by the necessary and sufficient conditions of the functional extreme. Thus the signals are obtained as a solution of a system of integral equations.A characteristic peculiarity of the form optimization problem is that the extremum is conditional, i.e. it is a problem of limitations. They determine a close area in the signal space where the signals searched for are found.

The paper presented examines and proposes a solution of the problem of synthesizing signals harmonized with the narrow-band channels of connection. The matching conditions between transmitted signals and communication channel are defined in terms of different limitations and the energy criterion.

2. FORMULATION OF THE PROBLEM OF HARMONIZING THE SIGNALS WITH THE CHANNEL OF CONNECTION

The problem of harmonizing with the channel of connection [3] can be formulated in such a way: within the class of signals $L_p[t_1, t_2]$ determined by limitations of

$$\left[\frac{1}{\Delta T}\int_{t_1}^{t_2}\left|s_j(t)\right|^p dt\right]^{1/p} \le S_p , \tag{1}$$

to find those $s_j(t)$, that maximize the functional of the quality

$$I = \left[\frac{1}{\Delta\tau}\int_{\tau_1}^{\tau_2}\left|x_i(t)\right|^m dt\right]^{1/m} . \tag{2}$$

Where: $x_i(t)$ – output signal;

$s_j(t)$ – input signal, i, j=1, 2, …, n ;

$t \in [t_1, t_2]$, $\Delta T = t_2 - t_1$;

$[\tau_1, \tau_2]$ is the interval of the observation on the output signal and $\Delta\tau = \tau_2 - \tau_1$.

It is assumed that the most common case in practice is the one with coincidence of the intervals under examination $[\tau_1, \tau_2]$ and signal existence $[t_1, t_2]$.

The possible cases are as follows:

- When m=1 – the functional of the quality is commensurable to the average rectified value of the output signal $x(t)$;
- When m→∞ - the functional is commensurable to the crest value of the output signal;
- When m=2 - the functional of the quality is commensurable to the energy of the output signal.

Expressing the output signal by the input one, the functional, which has to be maximized, takes the kind of:

$$I = \int_{t_1}^{t_2} \dots \int_{t_1}^{t_2} s(t_1)\dots s(t_v) H_v(t_1 \dots t_v) dt_1 \dots dt_v \quad . \quad (3)$$

where $H_v(t_1, \dots, t_v)$ is the nucleos of the functional and depends on the channel pulse characteristics *h(t)*.

The optimal signals are determined by the necessary and sufficient conditions of the functional extreme [1]. Thus the signals searched for are obtained as a solution of a system of integral equations.

The value of *p* in equation (1) depends on the kind of limitation imposed by the transmitter. The possible cases are as follows:

A. Limitation of the peak value of the signal ($p \to \infty$).

This limitation is grounded on the final length of the dynamic features of the transmitter output cascades or the non-linear nature of the active elements used with signal formation. It determines a class of optimal signals $L_\infty[t_1, t_2]$. Then functional (3) has its maximum [1] if the signal satisfies the requirement:

$$s(t_1) = S_\infty sign \int_{t_1}^{t_2} .. \int_{t_1}^{t_2} s(t_2)\dots s(t_v) H_v(t_1 \dots t_v) dt_2 \dots dt_v \quad (4)$$

i.e. harmonized signals *s(t)* belonging to $L_\infty[t_1, t_2]$ have the form of rectangular impulses of the kind shown in Fig.1.

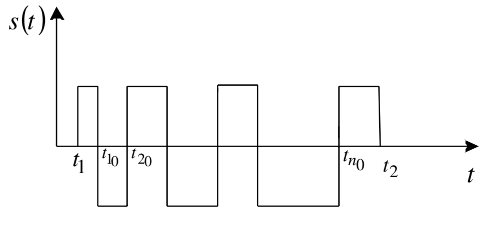

Fig.1. The signal belonging to $L_\infty[t_1, t_2]$

Kernel $H_v(t_1, \dots, t_v)$ defines the moment of the signal passing through zero, t_{i0} , i=1..N:

$$\int_{t_1}^{t_2} \dots \int_{t_1}^{t_2} s(t_2)\dots s(t_v) H_v(t_{i0} \dots t_v) dt_2 \dots dt_v = 0 \quad . \quad (5)$$

S_∞ is determined by the condition of the signal norm in $L_\infty[t_1, t_2]$.

For a linear Gauss's channel the equation (4) takes the kind of:

$$s(t_1) = S_\infty sign \;\; m_1\{h(t_{\max}, t_1)\}, m_1\{h(t_{\max}, t_{i0})\} = 0 \quad (6)$$

Where t_{max} is a moment where the output signal reaches maximum.

B. Limitation of the average rectified value of the signal (p=1).

Such a limitation can be observed with describing movable radio means where the sourse used to supply the transmitting device is overcharged earlier than the output series of the transmitter. The class of signals is $L_1[t_1, t_2]$. From the mathematical point of view, in the case of limiting the average rectified value of the signal, it is most convenient to use the inequality of Haldle [1] to work out the conditions necessary and sufficient for the maximum of functional (3). It can be proved, according to him, that in order to reach a maximum (3), signals *s(t)* have to satisfy the following condition:

$$s(t_1) = \sum_{i=1}^{N} a_i \delta(t_i - t_{i\max}) signF(t_1) \quad . \quad (7)$$

Where :

$$F(t_1) = \left| \int_{t_1}^{t_2} \dots \int_{t_1}^{t_2} s(t_2)\dots s(t_v) H_v(t_2 \dots t_v) dt_2 \dots dt_v \right| \quad . \quad (8)$$

- a_i are coefficients determined by the condition of the signal norm in $L_1[t_1, t_2]$:

$$\left[\frac{1}{\Delta T} \int_{t_1}^{t_2} |s_j(t)| dt \right] = S_1 \quad . \quad (9)$$

- $\{t_{i\,max}\}$, i=1..N, is a set of moments where function $F(t_1)$ reaches a maximum.

Having substituted (7) into (9) and taking into consideration the filtrating of δ- function, it is obtained that:

$$\sum_{i=1}^{N} a_i = \Delta T S_1 \quad . \quad (10)$$

Hence, the harmonized signals of class $L_1[t_1, t_2]$, with limited average rectified values that are determined by expression (7), represent a sequence of δ - pulses, supplied at the channel input at moment $\{t_{imax}\}$, when function (8) depending on the channel pulse characteristics has a maximum.

C. Limitation of the average square value of the signal (p=2).

The class of signals is $L_2[t_1, t_2]$. This is the Hilbert's linear area where the norm of the signal has been introduced by a scalar product and is commensurable to the energy examined.

According to the inequality of Haldle, that in order to reach a maximum (3), signals *s(t)* have to satisfy the following condition:

$$\frac{1}{\lambda}s(t_1)+\int_{t_1}^{t_2}.\int_{t_1}^{t_2}s(t_2)..s(t_v)H_v(t_1..t_v)dt_2..dt_v=0 \quad (11)$$

where λ is the eigen value of the nucleos:

$$H_v(t_1,..,t_v)=\int_{\max(t_1,t_2,\tau_1)}^{\tau_2} m_1\{h(t,t_1)...h(t,t_v)\}dt \ . \quad (12)$$

The solution of equation (11) for a channel with random coefficients of transfer and a delay for the most common case of fading [2] is:

$$s(t)=\sum_{i=1}^{N}a_i\delta(t-t_{\max}) \quad (13)$$

where a_i are coefficients that are determined by the condition of the signal norm in $L_2[t_1, t_2]$;

t_{max} is a moment where the coefficient of transfer reaches a maximum.

3. EFFICIENCY OF TRANSMITTING SIGNALS HARMONIZED WITH THE CHANNEL

The efficiency of transmitting signals harmonized with the channel will be assessed comparing them to signals in the kind of rectangular impulses transmitted along the linear narrow-band channel equivalent to a low-pass filter of impulse characteristics:

$$h(t)=\alpha.e^{-\alpha.t} \ . \quad (14)$$

Where α is the damping coefficient of the channel [2].

That can be done by the coefficient of efficiency expressed by the ratio of the coefficients with transmitting signals of both types:

$$\gamma^2=\frac{K_{opt}^2}{K_{rp}^2} \quad (15)$$

and showing what is the relation between the power of the signals at the channel output and the characteristic (14) in the both cases.

In the case when the signal is obtained as a solution of equation (7), the expression of the efficiency coefficient is as follows :

$$\gamma^2=\frac{\alpha\Delta T\left(1-e^{-2\alpha\Delta T}\right)}{2\left(1-\frac{3}{2\alpha\Delta T}+\frac{2e^{-\alpha\Delta T}}{\alpha\Delta T}-\frac{e^{-2\alpha\Delta T}}{2\alpha\Delta T}\right)} \ . \quad (16)$$

The graphic dependency $\gamma^2=f(\alpha\Delta T)$ is given in Fig.2. It is seen that the power of a signal in the kind of rectangular pulses (dashed line), of duration of $\tau=0{,}1\Delta T$, is lower at the channel output than that with transmitting a signal harmonized with the channel (the dense line).

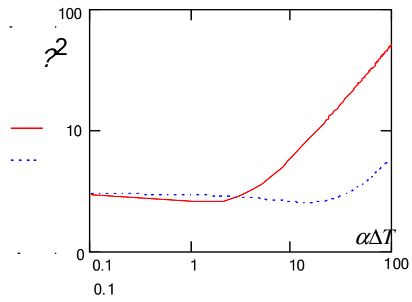

Fig. 2. The dependency $\gamma^2=f(\alpha\Delta T)$

In the case when the signal harmonized with the channel is obtained as a solution of equation (13) the following expression of the efficiency coefficient is as follows:

$$\gamma_1^2=\frac{\alpha.\Delta T}{2}\frac{1-e^{-2\alpha.\Delta T}}{\left(1-e^{-\alpha.\Delta T}\right)^2} \ . \quad (17)$$

The graphic dependency $\gamma_1^2=f(\alpha\Delta T)$ is given in Fig.3.

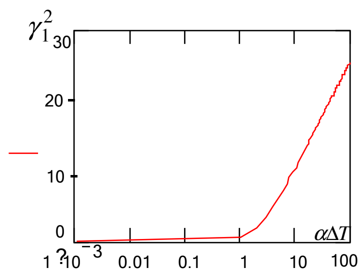

Fig. 3. The dependency $\gamma_1^2=f(\alpha\Delta T)$

At it can be seen in Fig.2 and Fig.3, the transmission of signals in the kind of δ - pulses, such as the harmonized ones with limited average rectified and average square values, results in a higher level of power at the channel output.

4. CONCLUSION

The coefficient of the impulse feature attenuation α is proportional to the channel frequency band and parameter αΔT characterizes its dispersive properties. With increasing αΔT, the energy efficiency of using harmonized signals also increases . From the results obtained it follows that with non-stationary channels one can achieve higher efficiency when short-time impulses are transmitted. If the non-stationary nature of the channel is known, the quality of connection can be considerably increased by selecting the most favorable moment of impulse transmitting.

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